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AN OPTIMIZATION PROCEDURE FOR WATER RESERVOIR PLANNING

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Abstract. A two-stage optimization procedure is developed. At the first stage the optimal values of irrigation area and installed power are determined subject to allowed deficits. The allowed deficits are given for irrigation and primary energy production. The demands for water supply are with high priority, and they are included in the constraints. At the second stage the problem of optimal control is solved applying dynamic programming. The algorithm at the first stage starts with maximal possible value for irrigation area S and installed power P. At the second stage the time series of inflow is used and the optimal control of reservoir is determined for given S and P from the first stage. The criterion for optimal control is maximum benefit. At the second stage S and P are modified subject to allowed deficit, and the second stage is run again, until the computed deficits become less or equal to the allowed. The optimization procedure may be terminated after a given number of iterations.

Key words and phrases: optimization, water, reservoir, hydropower, irrigation

1. INTRODUCTION

Water resources planning is concerned with the task of modifying the time and space availability of water for various purposes in order to accomplish certain goals. Planners are primarily concerned with providing quantities of water which can be assured, at some level of probability, to be available as needed to accomplish the basic goals. When we speak of meeting objectives, it is obvious that we are not dealing with the creation of perfection but rather with finding the best possible way of meeting goals within the limitations of resources. But what is "the best" and how do we find it? The answer is the aim of optimization.

A number of papers have already been published dealing with the optimization of multipurpose reservoirs. Dynamic programming is often applied to solve optimal control problem. There is an extensive literature on application of dynamic programming (DP) (Boudarel et al., 1971). An algorithm for the DP solution of the control problem of a single-purpose reservoir was given by Young (1967). The optimum design of a multipurpose reservoir is described by Hall (1964). A specific algorithm for optimal long-term control of multipurpose reservoir with direct and indirect users is described in the paper by Opricović and Đorđević (1976). Lately, discrete differential dynamic programming has been developed and applied to water resources system with more than three reservoirs (Yakowitz, 1982). The paper by Yeh (1985) reviewed the state-of-the-art of management models developed for reservoir operations. Much of the research in the area of reservoirs optimization has been based on deterministic optimization.

In this paper multipurpose water reservoir planning is considered. The purposes could be: irrigation, hydropower, water-supply or low flow augmentation. The matter of optimization in this paper are hydropower and irrigation, other purposes are considered with high priority and are included in the constraints.

2. PROBLEM STATEMENT

The water reservoir planning task is considered as the problem of determining the system parameters and the optimal control, according to given criterion and constraints. The scheme of the system is presented in the Figure 1.

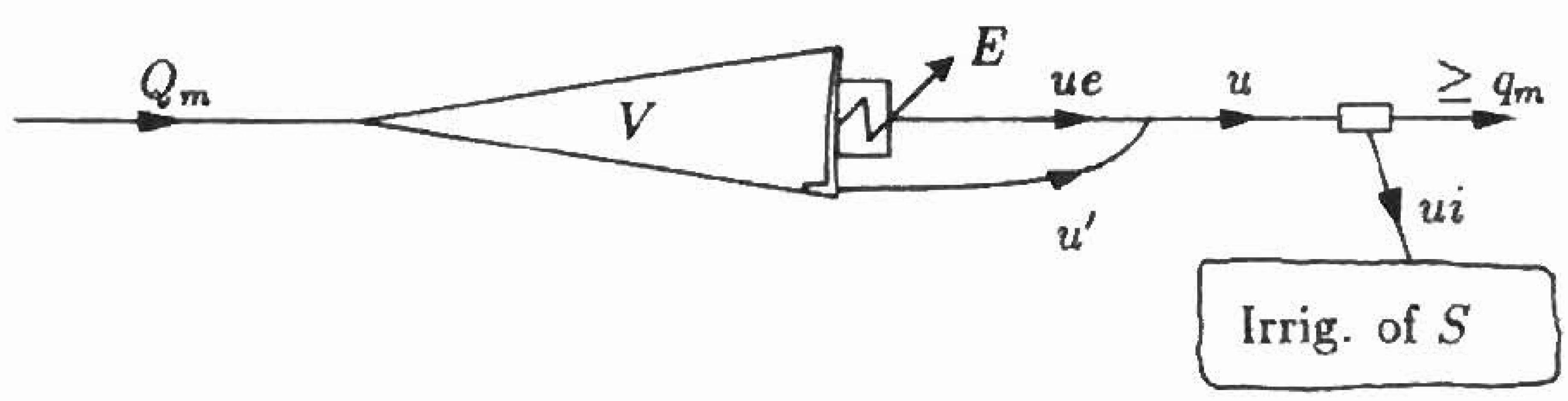

Figure 1. The scheme of the system

The system consists of river with deterministic flow, water reservoir, hydropower station, waterways, and water users (including irrigation area). The river flow (Q_m) is the inflow into the reservoir (V). The water from reservoir is delivered for energy production (E), for irrigation of the area S, and for downstream users (q_m).

The optimization task is to determine the installed power P for energy production and the area S for irrigation, as the "design" parameters (design problem), and optimal values of water delivered for each user in each month m, $m = 1, \dots, M$ (control problem).

The optimization problem is formulated as following:

$$\max_{S,P,u} \sum_{m=1}^{M} B_m(S, P, u) \qquad \text{subject to the constraints.}$$

This problem is decomposed into two problems:

I — Design problem

$$\max_{S,P} B^*(S, P) \qquad \text{subject to "design" constraints,}$$

II — Control problem

$$B^*(S,P) = \max_u \sum_{m=1}^{M} B_m(S,P,u) \qquad \text{subject to "control" constraints.}$$

This decomposition enables the development of a two-stage optimization procedure, described in the following section. The control problem is a "subproblem" of design problem.

In developing mathematical model for optimization the following main symbols are used:

- Q — inflow into water reservoir,
- V — water volume in the reservoir (state variable),
- u — water delivered from the reservoir (control variable),
- ue — outflow for energy production,
- ui — water used for irrigation,
- m — month (index in the time series),
- M — number of months within the time horizon,
- S — irrigation area (unknown)
- P — installed power capacity (unknown),
- B — benefit of water reservoir management,
- D_m — unit irrigation demand during the m-th month,
- h_m — number of operational hours within m-th month for primary (peak) energy production,
- $*$ — indicates the optimal value.

Dynamic variable without subscript (index m) indicates the time series, e.g., $ui = (ui_1, ui_2, \ldots, ui_M)$.

3. OPTIMIZATION METHODOLOGY

The two-stage optimization procedure is developed. At the first stage the optimal values of irrigation area S and installed power P are determined subject to allowed deficits. The allowed deficits are given for irrigation and primary energy production. At the second stage the problem of optimal control is solved applying dynamic programming.

The structure of the optimization algorithm RESOP (REServoir OPtimization) is presented in the Figure 2. At the first stage (SOLDEF) the optimal parameters S and P are determined by iterative procedure, satisfying the constraints of allowed deficits ("design" constraints). At the second stage (DYNPRO) the control problem is solved for the given values of S and P from the stage SOLDEF. The input data are given for: inflow (Q_m, $m = 1, \ldots, M$), allowed deficits, demands (D and h), unitary benefits, and system constants.

3.1. The First Optimization Stage — SOLDEF

At the first optimization stage S and P are unknown (parameters in "design" problem). The allowed deficits are given: ID — for irrigation, and ED — for primary energy production.

Computed deficits are:

$$id(S) = 100 \times \left(1 - \sum_{m=1}^{M} ui_m^* \Big/ \left(S \times \sum_{m=1}^{M} D_m\right)\right) \tag{1}$$

$$ed(P) = 100 \times \left(1 - \sum_{m=1}^{M} EP_m^* \Big/ \left(P \times \sum_{m=1}^{M} h_m\right)\right) \tag{2}$$

where

ui_m^* — is the optimal water volume for irrigation of S in the m-th month

EP_m^* — is the optimal value of primary energy

$$EP_m^* = \begin{cases} E_m^*, & \text{if } E_m^* < DE_m \\ DE_m, & \text{if } E_m^* \geq DE_m . \end{cases}$$

The optimal values ui^* and produced energy E^* are obtained at the second optimization stage by dynamic programming.

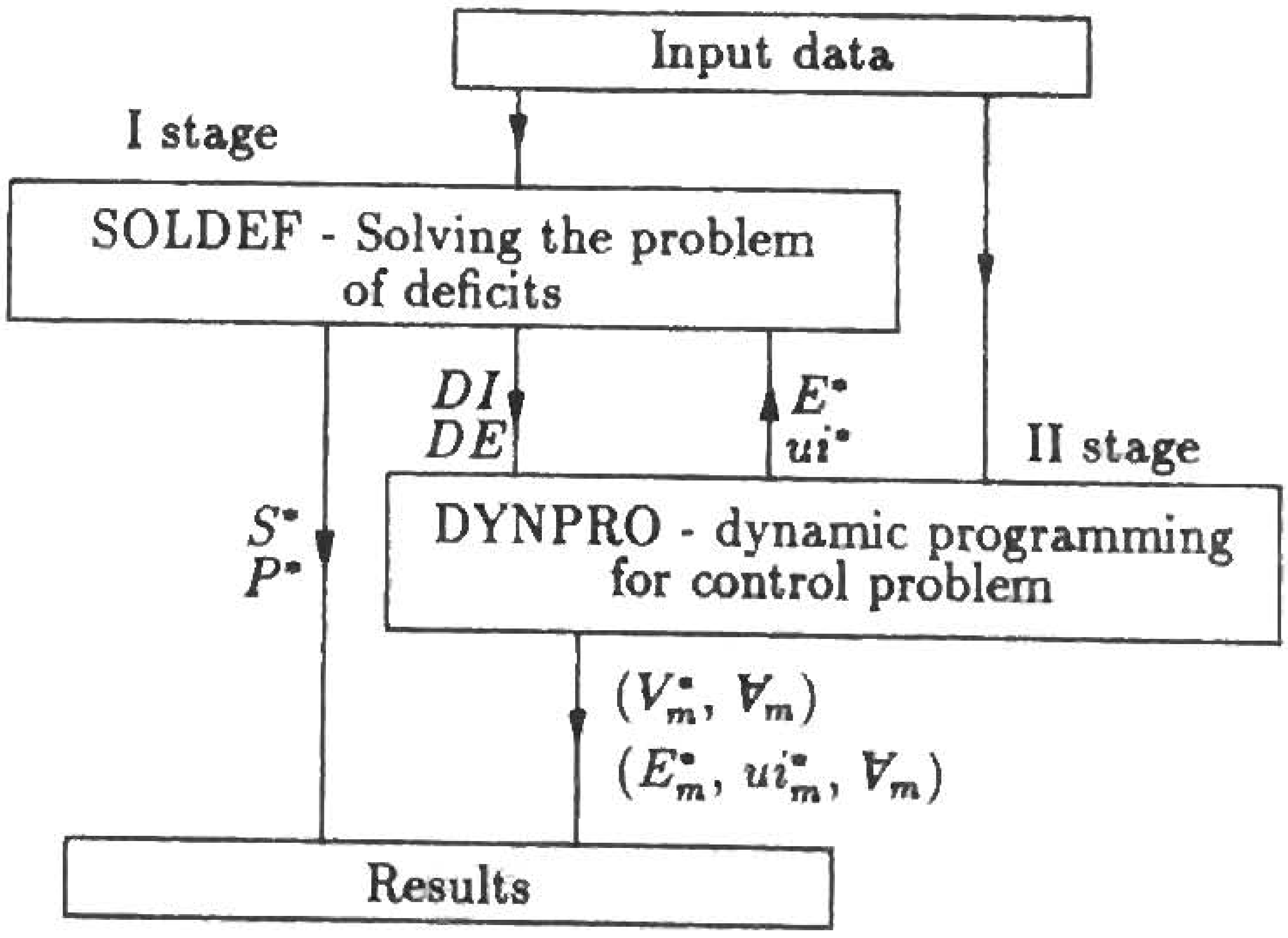

Fig. 2. The structure of the optimization algorithm

The algorithm at the first optimization stage (SOLDEF) starts with maximal possible (or initial) value for irrigation area S and for installed power P.

The demands in SOLDEF are

$$DI_m = S \times D_m \quad \text{and} \quad DE_m = P \times h_m, \qquad m = 1, \dots, M. \tag{3}$$

At the second optimization stage (DYNPRO) the time series of net inflow is used and the optimal control of reservoir is determined, with given DI and DE from SOLDEF. The results of optimal control E^* and ui^* are transferred to SOLDEF.

The deficits are computed by relations (1) and (2). Then the constraint $id \leq ID$ is checked. If it is not satisfied, the new value of S is computed from the following relation

$$S = \sum_{m=1}^{M} ui^*_m \Big/ \left((1 - ID/100) \times \sum_{m=1}^{M} D_m\right). \tag{4}$$

Also the constraint $ed \leq ED$ is checked. If this is not satisfied, the new value of P is computed from the relation:

$$P = \sum_{m=1}^{M} EP^*_m / \left((1 - ED/100) \times \sum_{m=1}^{M} h_m\right). \tag{5}$$

Each (next) iteration of optimization procedure consists of following steps:

1. Compute DI and DE by relation (3).
2. Subroutine DYNPRO (for optimal control).
3. Compute id by relation (1), and ed by (2).
4. Check the constraints: $id \leq ID$ and $ed \leq ED$. If both are satisfied, the optimization procedure terminates, otherwise continue with step 5.
5. Compute new value of S by relation (4), and P by (5). Continue with step 1.

The convergence of this iterative procedure is based on decreasing value of S and P (by relations (4) and (5)).

3.2. The Second Optimization Stage — DYNPRO

At the second optimization stage (subroutine DYNPRO) the problem of optimal control is solved applying dynamic programming. This approach is presented in the papers listed in references. The state variable is the water storage in reservoir at the end of months within the time horizon $(V_1, V_2, \ldots, V_M)$. The state equation is the relationship between state variable and control variable u.

The criterion for optimal control is maximum benefit. The criterion (objective) function has the following form:

$$F = \sum_{m=1}^{M} (BI_m + BE_m)$$

where:

$$BI_m = bi_m \times ui_m$$

$$BE_m = \begin{cases} bep_m \times E_m, & \text{if } E_m \leq DE_m \\ bep_m \times DE_m + bes_m \times (E_m - DE_m), & \text{if } E_m > DE_m \end{cases}$$

$$E_m = ce \times ue_m \times H_m.$$

and

- BI_m — denotes the irrigation benefit within the m-th month,
- BE_m — energy benefit
- bi — unitary irrigation benefit
- ui — water used for irrigation
- bep — unitary benefit of primary energy
- bes — unitary benefit of secondary energy
- DE — primary energy demand (determined by SOLDEF)
- ue — water used for energy production
- H_m — the power head
- ce — constant.

From the state equation it follows

$$u_m = V_{m-1} + Q_m - V_m$$

The conditions and constraints in the control space are:

$$\begin{aligned} &u_m \geq q_m \\ &ue_m + u'_m + Q_{\text{spill}} = u_m \\ &u'_m = 0 \quad \text{if} \quad \text{QEMAX}_m \geq DI_m + q_m \\ &u'_m \neq 0 \quad \text{if} \quad ue_m < ui_m + q_m \\ &ue_m + u'_m \geq ui_m + q_m \\ &ue_m \leq \text{QEMAX}_m \\ &ui_m \leq DI_m. \end{aligned} \tag{6}$$

QEMAX is the discharge capacity of turbines.

At the second optimization level the control problem is

$$\max_{(V_1,\dots,V_M)} F(V_1, V_2, \dots, V_M) \qquad \text{subject to set of constraints.}$$

By the principle of optimality the recurrence relation is derived in the following form

$$F_m(V_m) = \max_{V_{m-1}}[B_m(V_{m-1}, V_m) + F_{m-1}(V_{m-1})], \quad m = 2, \dots, M \tag{7}$$

$$F_1(V_1) = B_1(V_0, V_1)$$

where $B_m = BI_m + BE_m$ is the total benefit within the m-th month.

The forward dynamic programming algorithm is applied.

Let $V^+_{m-1}(V_m)$ denotes the value of V_{m-1} for which the maximum in (7) is obtained. All the $V^+_{m-1}(V_m)$, $m = 1, \dots, M$, are stored in the computer memory.

The optimal value V^*_M is determined by

$$F = \max_{V_M} F_M(V_M)$$

or, if it is given and feasible, $V_M^* = V_{M,\text{given}}$.

The optimal values of state of the reservoir are determined from the stored data by means of the following relation

$$V_m^* = V_m^+(V_{m+1}^*), \qquad m = M-1, M-2, \dots, 1. \tag{8}$$

Since the optimal trajectory of reservoir is determined $(V_1^*, V_2^*, \dots, V_M^*)$, the optimal control is determined as follows:

$$u_m^* = V_{m-1}^* + Q_m - V_m^*$$

$$ui_m^* = \begin{cases} 0 & \text{if } u_m^* \le q_m \text{ or } DI_m = 0, \\ u_m^* + q_m & \text{if } q_m < u_m^* < DI_m + q_m \\ DI_m & \text{if } u_m^* \ge DI_m + q_m \end{cases}$$

$$ue_m^* = \begin{cases} 0 & \text{if } V_m^* \le \text{WEMIN or } V_{m-1}^* \le \text{WEMIN} \\ u_m^* & \text{if } 0 \le u_m^* < \text{QEMAX}_m \\ \text{QEMAX}_m & \text{if } u_m^* \ge \text{QEMAX}_m \end{cases}$$

$$E_m^* = ce \times ue_m^* \times H(0.5 \times (V_{m-1}^* + V_m^*) - HTW).$$

WEMIN is the minimal state for hydropower, HTW — tailwater.

The optimization results could be statistically analysed.

4. APPLICATION

This optimization model was developed in order to solve practical problems of multipurpose reservoirs design. Some results are presented only to illustrate the application of this model. Testing was performed with the data of reservoir on River San Huan in Argentina.

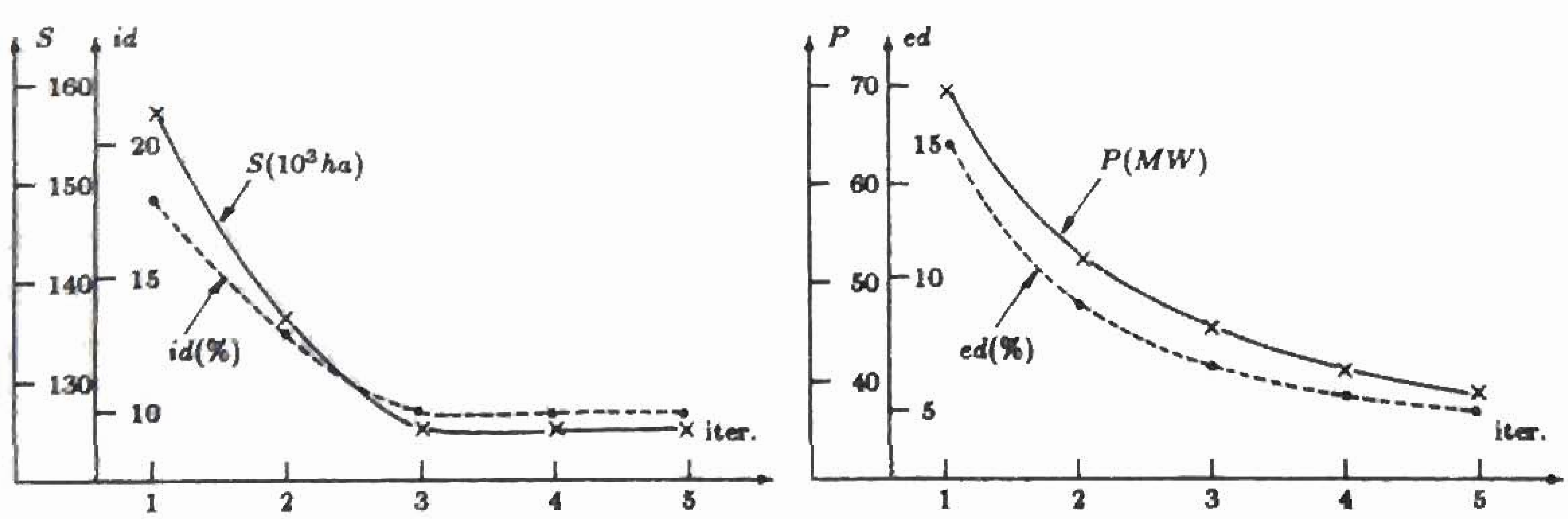

Fig. 3. The convergence of the results

The results are presented in Figures 3 and 4. The Fig. 3 shows the convergence of the results in five iterations. The allowed irrigation deficit is reached after three iterations, and the allowed deficit for primary energy production after five iterations.

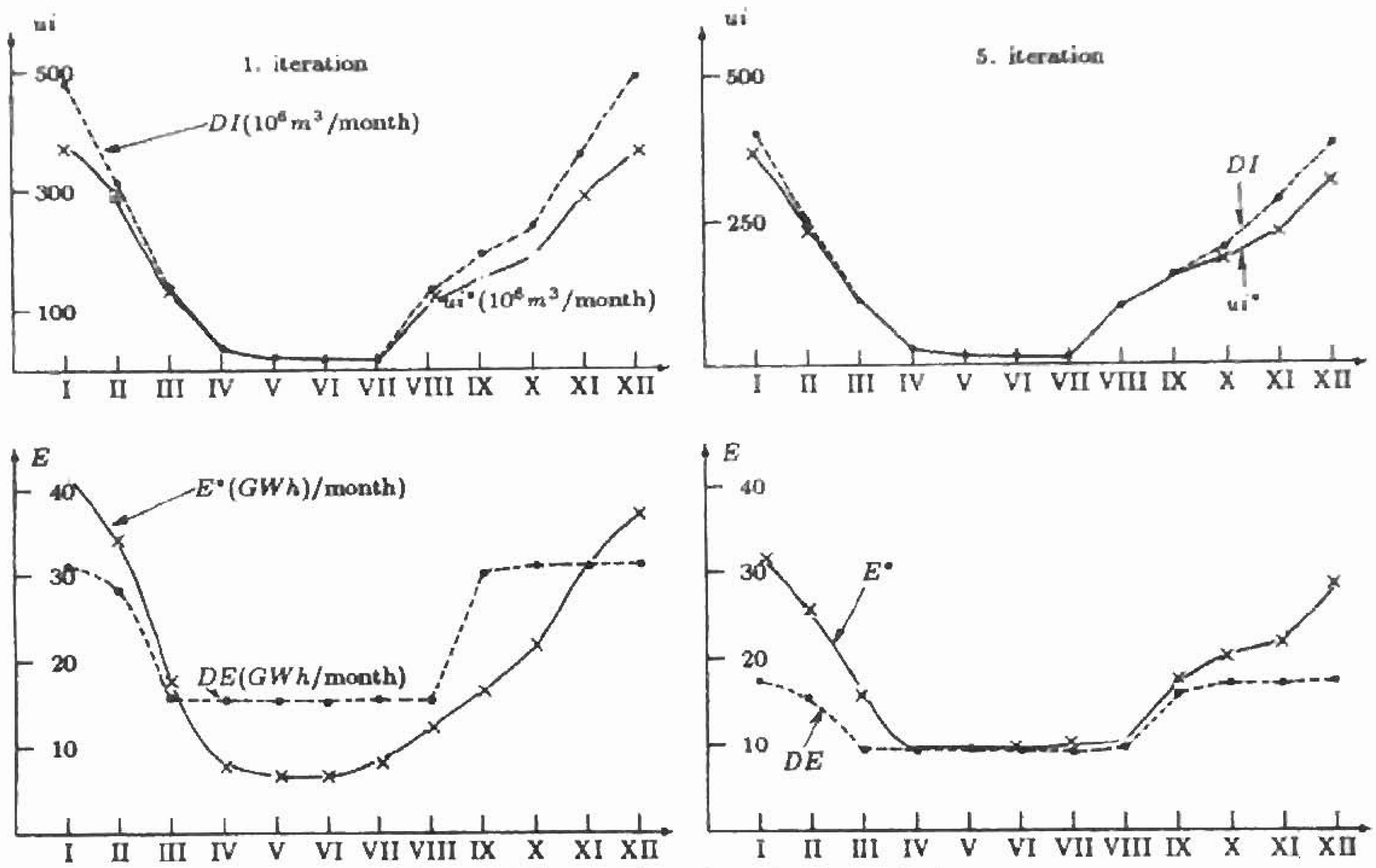

Fig. 4. The results of optimal control

The Fig. 4 shows the results of optimal control. The results are obtain for given time series (20 years) of inflow. The average monthly values of water used for irrigation ui^* and produced energy E^* are presented. The Fig. 4 shows that there is unallowed deficit after first iteration. After five iterations the deficit for irrigation and energy production are decreased to the allowed deficit. The secondary energy has to be produced by the water released for irrigation (Fig. 1).

CONCLUSION

The presented optimization method is developed for the tasks of water reservoir planning. The optimal value of irrigation area and installed power are determined subject to allowed deficits, and with optimal control. The computer program is written in FORTRAN. It could be used for the following optimization tasks:

— multipurpose water reservoir planning (main task)

— water reservoir management (with given parameters S and P)

— water reservoir design (for variants of reservoir capacity).

The optimization method is based on the new approach to multipurpose water reservoir planning and on known algorithm for optimal control applying dynamic programming.

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